INSTRUCTOR'S SOLUTIONS MANUAL

BEVERLY FUSFIELD

CALCULUS & ITS APPLICATIONS

and

BRIEF CALCULUS & ITS APPLICATIONS

THIRTEENTH EDITION

Larry J. Goldstein
Goldstein Educational Technologies

David C. Lay
University of Maryland

David I. Schneider
University of Maryland

Nakhlé Asmar
University of Missouri

PEARSON

Boston Columbus Indianapolis New York San Francisco Upper Saddle River
Amsterdam Cape Town Dubai London Madrid Milan Munich Paris Montreal Toronto
Delhi Mexico City São Paulo Sydney Hong Kong Seoul Singapore Taipei Tokyo

Reproduced by Pearson from electronic files supplied by the author.

Publishing as Pearson, 75 Arlington Street, Boston, MA 02116.

ISBN-13: 978-0-321-87879-3
ISBN-10: 0-321-87879-5

www.pearsonhighered.com

PEARSON

CONTENTS

Chapter 0 Functions 1

Chapter 1 The Derivative 26

Chapter 2 Applications of the Derivative 77

Chapter 3 Techniques of Differentiation 115

Chapter 4 The Exponential and Natural Logarithmic Functions 141

Chapter 5 Applications of the Exponential and Natural Logarithm Functions 168

Chapter 6 The Definite Integral 182

Chapter 7 Functions of Several Variables 215

Chapter 8 The Trigonometric Functions 245

Chapter 9 Techniques of Integration 263

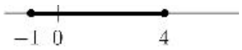
Chapter 10 Differential Equations 302

Chapter 11 Taylor Polynomials and Infinite Series 334

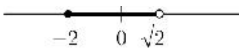
Chapter 12 Probability and Calculus 356

Chapter 0 Functions

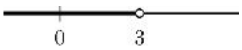
0.1 Functions and Their Graphs

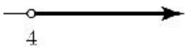
1. 

2. 

3. 

4. 

5. 

6. 

7. $[2, 3)$ **8.** $\left(-1, \dfrac{3}{2}\right)$

9. $[-1, 0)$ **10.** $[-1, 8)$

11. $(-\infty, 3)$ **12.** $\left[\sqrt{2}, \infty\right)$

13. $f(x) = x^2 - 3x$

$f(0) = 0^2 - 3(0) = 0$

$f(5) = 5^2 - 3(5) = 25 - 15 = 10$

$f(3) = 3^2 - 3(3) = 9 - 9 = 0$

$f(-7) = (-7)^2 - 3(-7) = 49 + 21 = 70$

14. $f(x) = x^3 + x^2 - x - 1$

$f(1) = 1^3 + 1^2 - 1 - 1 = 0$

$f(-1) = (-1)^3 + (-1)^2 - (-1) - 1 = 0$

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 1 = -\frac{9}{8}$

$f(a) = a^3 + a^2 - a - 1$

15. $g(t) = t^3 - 3t^2 + t$

$g(2) = 2^3 - 3(2)^2 + 2 = 8 - 12 + 2 = -2$

$$g\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) = -\frac{1}{8} - \frac{3}{4} - \frac{1}{2} = -\frac{11}{8}$$

$$g\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - 3\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right) = \frac{8}{27} - \frac{12}{9} + \frac{2}{3} = -\frac{10}{27} \approx -.37037$$

$g(a) = a^3 - 3a^2 + a$

16. $h(s) = \dfrac{s}{(1+s)}$

$$h\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\left(1+\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

$$h\left(-\frac{3}{2}\right) = \frac{-\frac{3}{2}}{1+\left(-\frac{3}{2}\right)} = \frac{-\frac{3}{2}}{-\frac{1}{2}} = 3$$

$$h(a+1) = \frac{a+1}{1+(a+1)} = \frac{a+1}{a+2}$$

17. $f(x) = x^2 - 2x$

$f(a+1) = (a+1)^2 - 2(a+1)$
$= (a^2 + 2a + 1) - 2a - 2 = a^2 - 1$

$f(a+2) = (a+2)^2 - 2(a+2)$
$= (a^2 + 4a + 4) - 2a - 4 = a^2 + 2a$

18. $f(x) = x^2 + 4x + 3$

$f(a-1) = (a-1)^2 + 4(a-1) + 3$
$= (a^2 - 2a + 1) + (4a - 4) + 3$
$= a^2 + 2a$

$f(a-2) = (a-2)^2 + 4(a-2) + 3$
$= (a^2 - 4a + 4) + (4a - 8) + 3$
$= a^2 - 1$

19. **a.** $f(0)$ represents the number of laptops sold in 2010.

b. $f(6) = 150 + 2(6) + 6^2$
$= 150 + 12 + 36 = 198$

20. $R(x) = \dfrac{100x}{b+x},\ x \geq 0$

a. $b = 20, x = 60$

$$R(60) = \frac{100(60)}{20+60} = 75$$

The solution produces a 75% response.

b. If $R(50) = 60$, then

$$60 = \frac{100(50)}{b+50}$$

$$60b + 3000 = 5000$$

$$b = \frac{100}{3}$$

This particular frog has a positive constant of $33.\bar{3}$.

21. $f(x)=\dfrac{8x}{(x-1)(x-2)}$
all real numbers such that $x \neq 1, 2$ or
$(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

22. $f(t)=\dfrac{1}{\sqrt{t}}$
all real numbers such that $t > 0$ or $(0,\infty)$

23. $g(x)=\dfrac{1}{\sqrt{3-x}}$
all real numbers such that $x < 3$ or $(-\infty,-3)$

24. $g(x)=\dfrac{4}{x(x+2)}$
all real numbers such that $x \neq 0, -2$ or
$(-\infty,-2)\cup(-2,0)\cup(0,\infty)$

25. $f(x)=\dfrac{3x-5}{x^2+x-6}$
The denominator is zero for $x = -3$ and $x = 2$, so the domain consists of all real numbers such that $x \neq -3, 2$ or
$(-\infty, -3)\cup(-3, 2)\cup(2,\infty)$.

26. $f(x)=\dfrac{1}{3x^2+1}$
The domain consists of all real numbers, or
$(-\infty, \infty)$.

27. $f(x)=\sqrt{2x+7}+\sqrt{x}$
The domain consists of all real numbers greater than or equal to 0, or $[0,\infty)$.

28. $f(x)=\dfrac{\sqrt{2x+1}}{\sqrt{1-x}}$
The denominator is greater than 0 for all $x < 1$, and the numerator is defined for all $x \geq -\dfrac{1}{2}$.
Thus, the domain is $-\dfrac{1}{2} \leq x < 1$, or $\left[-\dfrac{1}{2}, 1\right)$.

29. function
30. not a function
31. not a function
32. not a function
33. not a function
34. function

35. $f(0)=1;\ \ f(7)=-1$

36. $f(2)=3;\ \ f(-1)=0$

37. positive
38. negative

39. $[-1, 3]$
40. $-1, 5, 9$

41. $(-\infty, -1]\cup[5, 9]$
42. $[-1,5]\cup[9,\infty]$

43. $f(1)\approx .03;\ \ f(5)\approx .037$

44. $f(6)\approx .03$

45. $[0, .05]$
46. $t \approx 3$

47. $f(x)=\left(x-\dfrac{1}{2}\right)(x+2)$
$f(3)=\left(3-\dfrac{1}{2}\right)(3+2)=\dfrac{25}{2}$
Thus, (3, 12) is not on the graph.

48. $f(x) = x(5 + x)(4 - x)$
$f(-2) = -2(5 + (-2))(4 - (-2)) = -36$
So $(-2, 12)$ is not on the graph.

49. $g(x)=\dfrac{3x-1}{x^2+1}$
$g\left(\dfrac{1}{3}\right)=\dfrac{3\left(\frac{1}{2}\right)-1}{\left(\frac{1}{2}\right)^2+1}=\dfrac{\frac{1}{2}}{\frac{5}{4}}=\dfrac{2}{5}$
So $\left(\dfrac{1}{2}, \dfrac{2}{5}\right)$ is on the graph.

50. $g(x)=\dfrac{(x^2+4)}{(x+2)}$
$g\left(\dfrac{2}{3}\right)=\dfrac{\left(\frac{2}{3}\right)^2+4}{\frac{2}{3}+2}=\dfrac{\frac{40}{9}}{\frac{8}{3}}=\dfrac{5}{3}$
So $\left(\dfrac{2}{3}, \dfrac{5}{3}\right)$ is on the graph.

51. $f(x)=x^3$
$f(a+1)=(a+1)^3$

52. $f(x)=\left(\dfrac{5}{x}\right)-x$
$f(2+h)=\dfrac{5}{(2+h)}-(2+h)$
$=\dfrac{5-(2+h)^2}{(2+h)}=\dfrac{1-4h-h^2}{2+h}$

53. $f(x)=\begin{cases}\sqrt{x} & \text{for } 0 \leq x < 2\\ 1+x & \text{for } 2 \leq x \leq 5\end{cases}$
$f(1)=\sqrt{1}=1;\ \ f(2)=1+2=3$
$f(3)=1+3=4$

54. $f(x)=\begin{cases}\dfrac{1}{x} & \text{for } 1\le x\le 2\\ x^2 & \text{for } 2<x\end{cases}$

$f(1)=\dfrac{1}{1}=1$; $f(2)=\dfrac{1}{2}$

$f(3)=3^2=9$

55. $f(x)=\begin{cases}\pi x^2 & \text{for } x<2\\ 1+x & \text{for } 2\le x\le 2.5\\ 4x & \text{for } 2.5<x\end{cases}$

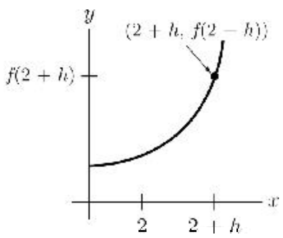
$f(1)=\pi(1)^2=\pi$
$f(2)=1+2=3$
$f(3)=4(3)=12$

56. $f(x)=\begin{cases}\dfrac{3}{4-x} & \text{for } x<2\\ 2x & \text{for } 2\le x<3\\ \sqrt{x^2-5} & \text{for } 3\le x\end{cases}$

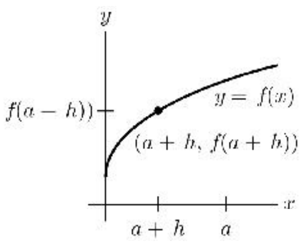
$f(1)=\dfrac{3}{4-1}=1$
$f(2)=2(2)=4$
$f(3)=\sqrt{3^2-5}=\sqrt{4}=2$

57. $f(x)=\begin{cases}.06x & \text{for } 50\le x\le 300\\ .02x+12 & \text{for } 300<x\le 600\\ .015x+15 & \text{for } 600<x\end{cases}$

58.



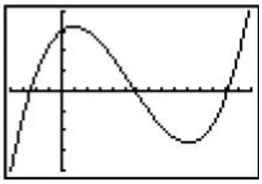
59.



60. Entering $\mathbf{Y_1 = 1/X + 1}$ will graph the function $f(x)=\dfrac{1}{x}+1$. In order to graph the function $f(x)=\dfrac{1}{x+1}$, you need to include parentheses in the denominator: $\mathbf{Y_1 = 1/(X + 1)}$.

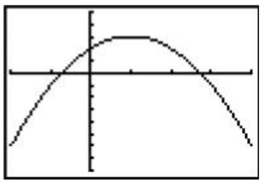
61. Entering **$\mathbf{Y_1}$ = X ^ 3 / 4** will graph the function $f(x)=\dfrac{x^3}{4}$. In order to graph the function $y=x^{3/4}$, you need to include parentheses in the exponent: **$\mathbf{Y_1}$ = X ^ (3/4)**.

62. Y_1 = X^3 – 33X^2 +120X+1500



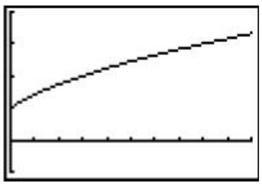
[–8, 30] by [–2000, 2000]

63. Y_1 = –X^2+2x+2



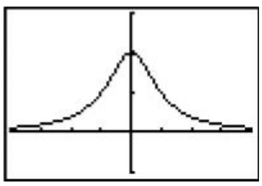
[–2, 4] by [–8, 5]

64. Y_1 = (X+1)^(1/2)



[0, 10] by [–1, 4]

65. Y_1 = 1/(X^2+1)

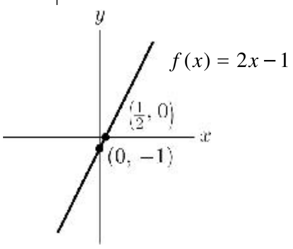


[–4, 4] by [–.5, 1.5]

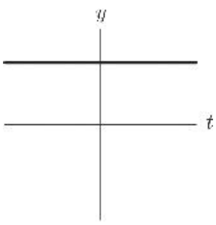
0.2 Some Important Functions

1. $y = 2x - 1$

x	y
1	1
0	–1
–1	–3

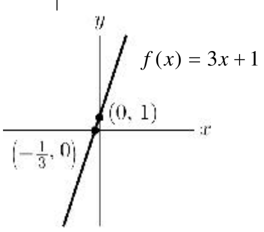


2. $y = 3$



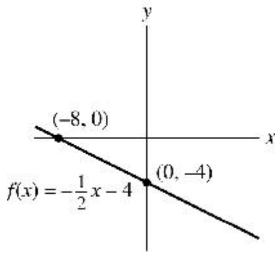
3. $y = 3x + 1$

x	y
1	4
0	1
–1	–2



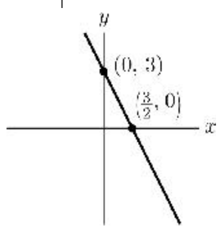
4. $y = -\frac{1}{2}x - 4$

x	y
2	–5
0	–4
–2	–3

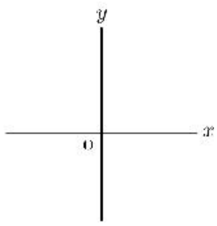


5. $x = -2x + 3$

x	y
–1	5
0	3
1	1

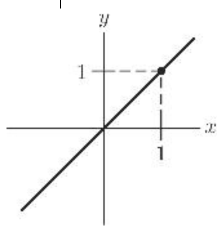


6. $x = 0$



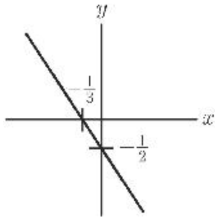
7. $x - y = 0$

x	y
1	1
0	0
–1	–1



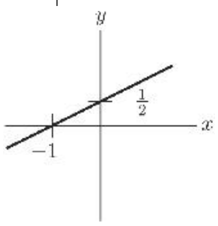
8. $3x + 2y = -1$

x	y
3	–5
1	–2
–3	4



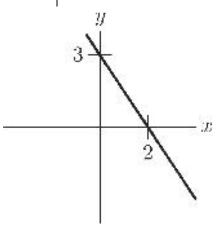
9. $x = 2y - 1$

x	y
3	2
1	1
–3	–1



10. $\frac{x}{2} + \frac{y}{3} = 1$

x	y
2	0
0	3
–2	6



11. $f(x) = 9x + 3$

$f(0) = 9(9) + 3 = 3$

The y-intercept is (0, 3).

$9x + 3 = 0 \Rightarrow 9x = -3 \Rightarrow x = -\frac{1}{3}$

The x-intercept is $\left(-\frac{1}{3}, 0\right)$.

12. $f(x) = -\frac{1}{2}x - 1$

$f(0) = -\frac{1}{2}(0) - 1 = -1$

The y-intercept is (0, –1).

$-\frac{1}{2}x - 1 = 0 \Rightarrow -\frac{1}{2}x = 1 \Rightarrow x = -2$

The x-intercept is (–2, 0).

13. $f(x) = 5$

The y-intercept is (0, 5).

There is no x-intercept.

14. $f(x) = 14$

The y-intercept is (0, 14).

There is no x-intercept.

15. $x - 5y = 0$

$0 - 5y = 0 \Rightarrow y = 0$

The x- and y-intercept is (0, 0).

16. $2 + 3x = 2y$

$2 + 3(0) = 2y \Rightarrow y = 1$

The y-intercept is (0, 1).

$2 + 3x = 2(0) \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$

The x-intercept is $\left(-\frac{2}{3}, 0\right)$.

17. $f(x) = \left(\frac{K}{V}\right)x + \frac{1}{V}$

a. $f(x) = .2x + 50$

We have $\frac{K}{V} = .2$ and $\frac{1}{V} = 50$. If $\frac{1}{V} = 50$, then $V = \frac{1}{50}$. Now, $\frac{K}{V} = .2$ implies $\frac{K}{\frac{1}{50}} = .2$, so $K = \frac{1}{5} \cdot \frac{1}{50} = \frac{1}{250}$.

b. $y = \left(\frac{K}{V}\right)x + \frac{1}{V}$, $\left(\frac{K}{V}\right) \cdot 0 + \frac{1}{V} = \frac{1}{V}$, so the y-intercept is $\left(0, \frac{1}{V}\right)$.

Solving $\left(\frac{K}{V}\right)x + \frac{1}{V} = 0$, we get $\frac{K}{V}x = -\frac{1}{V} \Rightarrow x = -\frac{1}{K}$, so the x-intercept is $\left(-\frac{1}{K}, 0\right)$.

18. From 17(b), $\left(-\frac{1}{K}, 0\right)$ is the x-intercept. From the experimental data, (–500, 0) is also the x-intercept. Thus $-\frac{1}{K} = -500,\ K = \frac{1}{500}$.
Again from 17(b), $\left(0, \frac{1}{V}\right)$ is the y-intercept. From the experimental data, (0, 60) is also the y-intercept. Thus $\frac{1}{V} = 60,\ V = \frac{1}{60}$.

19. **a.** Cost is \$(24 + 200(.25)) = \$74.

b. $f(x) = .25x + 24$

20. Let x be the volume of gas (in thousands of cubic feet) extracted.
$f(x) = 5000 + .10x$

21. Let x be the number of days of hospital confinement.
$f(x) = 700x + 1900$

22. $6x - 40 = 350 \Rightarrow x = 65$ mph

23. $f(x) = \frac{50x}{105 - x},\ 0 \le x \le 100$
From example 6, we know that $f(70) = 100$. The cost to remove 75% of the pollutant is
$f(75) = \frac{50 \cdot 75}{105 - 75} = 125.$
The cost of removing an extra 5% is \$125 – \$100 = \$25 million. To remove the final 5% the cost is
$f(100) - f(95) = 1000 - 475 = \$525$ million. This costs 21 times as much as the cost to remove the next 5% after the first 70% is removed.

24. **a.** $f(85) = \frac{20(85)}{102 - 85} = \$100$ million

b. $f(100) - f(95) = 1000 - 271.43 \approx \$728.57$ million

25. $y = 3x^2 - 4x$
$a = 3, b = -4, c = 0$

26. $y = \frac{x^2 - 6x + 2}{3} = \frac{1}{3}x^2 - 2x + \frac{2}{3}$
$a = \frac{1}{3},\ b = -2,\ c = \frac{2}{3}$

27. $y = 3x - 2x^2 + 1$
$a = -2, b = 3, c = 1$

28. $y = 3 - 2x + 4x^2$
$a = 4, b = -2, c = 3$

29. $y = 1 - x^2$
$a = -1, b = 0, c = 1$

30. $y = \frac{1}{2}x^2 + \sqrt{3}x - \pi$
$a = \frac{1}{2},\ b = \sqrt{3},\ c = -\pi$

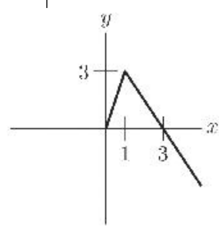
31. $f(x) = \begin{cases} 3x & \text{for } 0 \le x \le 1 \\ \frac{9}{2} - \frac{3}{2}x & \text{for } x > 1 \end{cases}$

$0 \le x \le 1$

x	$f(x) = 3x$
0	0
1	3

$x > 1$

x	$f(x) = \frac{9}{2} - \frac{3}{2}x$
2	$\frac{3}{2}$
3	0



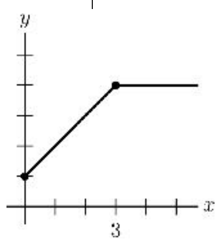
32. $f(x) = \begin{cases} 1 + x & \text{for } x \le 3 \\ 4 & \text{for } x > 3 \end{cases}$

$x \le 3$

x	$f(x) = 1 + x$
0	1
3	4

$x > 3$

x	$f(x) = 4$
4	4
5	4



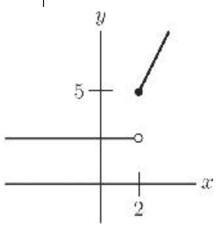
33. $f(x)=\begin{cases}3 & \text{for } x<2\\ 2x+1 & \text{for } x\geq 2\end{cases}$

$x<2$

x	$f(x)=3$
1	3
0	3

$x\geq 2$

x	$f(x)=2x+1$
2	5
3	7



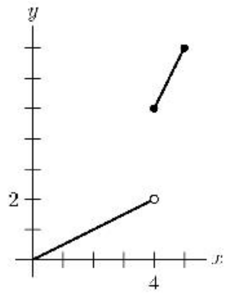
34. $f(x)=\begin{cases}\frac{1}{2}x & \text{for } 0\leq x<4\\ 2x-3 & \text{for } 4\leq x\leq 5\end{cases}$

$0\leq x<4$

x	$f(x)=\frac{1}{2}x$
0	0
2	1
3	$\frac{3}{2}$

$4\leq x\leq 5$

x	$f(x)=2x-3$
4	5
5	7



35. $f(x)=\begin{cases}4-x & \text{for } 0\leq x<2\\ 2x-2 & \text{for } 2\leq x<3\\ x+1 & \text{for } x\geq 3\end{cases}$

$0\leq x<2$

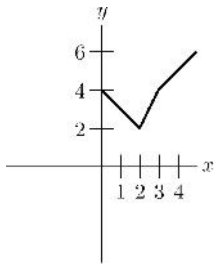
x	$f(x)=4-x$
0	4
1	3

$2\leq x<3$

x	$f(x)=2x-2$
2	2
$\frac{5}{2}$	3

$x\geq 3$

x	$f(x)=x+1$
3	4
4	5



36. $f(x)=\begin{cases}4x & \text{for } 0\leq x<1\\ 8-4x & \text{for } 1\leq x<2\\ 2x-4 & \text{for } x\geq 2\end{cases}$

$0\leq x<1$

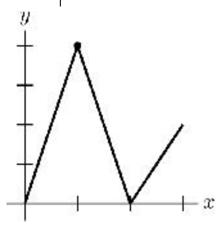
x	$f(x)=4x$
0	0
$\frac{1}{2}$	2

$1\leq x<2$

x	$f(x)=8-4x$
1	4
$\frac{3}{2}$	2

$x\geq 2$

x	$f(x)=2x-4$
2	0
3	2



37. $f(x)=x^{100},\ x=-1$

$f(-1)=(-1)^{100}=1$

38. $f(x)=x^5,\ x=\frac{1}{2}$

$f\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^5=\frac{1}{32}$

39. $f(x)=|x|,\ x=10^{-2}$

$f(10^{-2})=|10^{-2}|=10^{-2}$

40. $f(x)=|x|,\ x=\pi$

$f(\pi)=|\pi|=\pi$

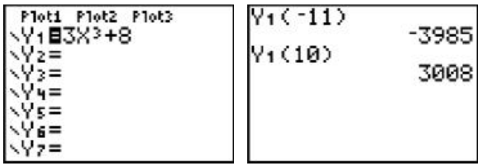
41. $f(x)=|x|,\ x=-2.5$

$f(-2.5)=|-2.5|=2.5$

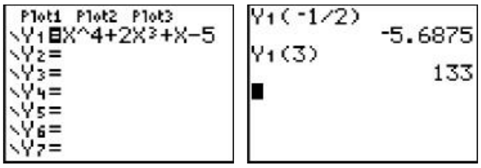
42. $f(x)=|x|,\ x=-\dfrac{2}{3}$

$$f\left(-\frac{2}{3}\right)=\left|-\frac{2}{3}\right|=\frac{2}{3}$$

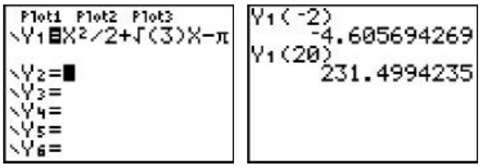
43.



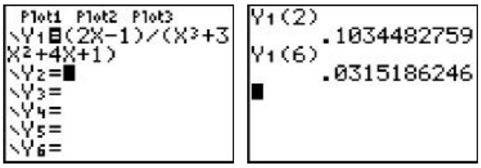
44.



45.



46.



0.3 The Algebra of Functions

1. $f(x)+g(x)=(x^2+1)+9x=x^2+9x+1$

2. $f(x)-h(x)=(x^2+1)-(5-2x^2)=3x^2-4$

3. $f(x)g(x)=(x^2+1)(9x)=9x^3+9x$

4. $g(x)h(x)=(9x)(5-2x^2)=45x-18x^3$

5. $\dfrac{f(t)}{g(t)}=\dfrac{t^2+1}{9t}=\dfrac{t^2}{9t}+\dfrac{1}{9t}=\dfrac{t}{9}+\dfrac{1}{9t}=\dfrac{t^2+1}{9t}$

6. $\dfrac{g(t)}{h(t)}=\dfrac{9t}{5-2t^2}$

7. $\dfrac{2}{x-3}+\dfrac{1}{x+2}=\dfrac{2(x+2)+(x-3)}{(x-3)(x+2)}=\dfrac{3x+1}{x^2-x-6}$

8. $\dfrac{3}{x-6}+\dfrac{-2}{x-2}=\dfrac{3(x-2)+(-2)(x-6)}{(x-6)(x-2)}=\dfrac{x+6}{x^2-8x+12}$

9. $\dfrac{x}{x-8}+\dfrac{-x}{x-4}=\dfrac{x(x-4)+(-x)(x-8)}{(x-8)(x-4)}=\dfrac{4x}{x^2-12x+32}$

10. $\dfrac{-x}{x+3}+\dfrac{x}{x+5}=\dfrac{(-x)(x+5)+x(x+3)}{(x+3)(x+5)}=\dfrac{-2x}{x^2+8x+15}$

11. $\dfrac{x+5}{x-10}+\dfrac{x}{x+10}=\dfrac{(x+5)(x+10)+x(x-10)}{(x-10)(x+10)}=\dfrac{2x^2+5x+50}{x^2-100}$

12. $\dfrac{x+6}{x-6}+\dfrac{x-6}{x+6}=\dfrac{(x+6)(x+6)+(x-6)(x-6)}{(x-6)(x+6)}=\dfrac{2x^2+72}{x^2-36}$

13. $\dfrac{x}{x-2}-\dfrac{5-x}{5+x}=\dfrac{x(5+x)-(5-x)(x-2)}{(x-2)(5+x)}=\dfrac{2x^2-2x+10}{x^2+3x-10}$

14. $\dfrac{t}{t-2}-\dfrac{t+1}{3t-1}=\dfrac{t(3t-1)-(t-2)(t+1)}{(t-2)(3t-1)}=\dfrac{2t^2+2}{3t^2-7t+2}$

15. $\dfrac{x}{x-2}\cdot\dfrac{5-x}{5+x}=\dfrac{-x^2+5x}{x^2+3x-10}$

16. $\dfrac{5-x}{5+x}\cdot\dfrac{x+1}{3x-1}=\dfrac{-x^2+4x+5}{3x^2+14x-5}$

17. $\dfrac{\frac{x}{x-2}}{\frac{5-x}{5+x}}=\dfrac{x}{x-2}\cdot\dfrac{5+x}{5-x}=\dfrac{x^2+5x}{-x^2+7x-10}$

18. $\dfrac{\frac{s+1}{3s-1}}{\frac{s}{s-2}}=\dfrac{s+1}{3s-1}\cdot\dfrac{s-2}{s}=\dfrac{s^2-s-2}{3s^2-s}$

19. $\dfrac{x+1}{(x+1)-2}\cdot\dfrac{5-(x+1)}{5+(x+1)}=\dfrac{x+1}{x-1}\cdot\dfrac{-x+4}{6+x}=\dfrac{-x^2+3x+4}{x^2+5x-6}$

20. $\dfrac{x+2}{(x+2)-2}+\dfrac{5-(x+2)}{5+(x+2)}$
$=\dfrac{x+2}{x}+\dfrac{3-x}{x+7}$
$=\dfrac{(x+2)(x+7)+(3-x)(x)}{x(x+7)}=\dfrac{12x+14}{x^2+7x}$

21. $\dfrac{\dfrac{5-(x+5)}{5+(x+5)}}{\dfrac{x+5}{(x+5)-2}}=\dfrac{5-(x+5)}{5+(x+5)}\cdot\dfrac{(x+5)-2}{x+5}$
$=\dfrac{-x}{10+x}\cdot\dfrac{x+3}{x+5}$
$=\dfrac{-x^2-3x}{x^2+15x+50}$

22. $\dfrac{\frac{1}{t}}{\frac{1}{t}-2}=\dfrac{1}{t}\cdot\dfrac{t}{1-2t}=\dfrac{1}{1-2t}$, $t\neq 0$

23. $\dfrac{5-\frac{1}{u}}{5+\frac{1}{u}}=\dfrac{5u-1}{u}\cdot\dfrac{u}{5u+1}=\dfrac{5u-1}{5u+1}$, $u\neq 0$

24. $\dfrac{\frac{1}{x^2}+1}{3\left(\frac{1}{x^2}\right)-1}=\dfrac{1+x^2}{x^2}\cdot\dfrac{x^2}{3-x^2}=\dfrac{1+x^2}{3-x^2}, x\neq 0$

25. $f\left(\dfrac{x}{1-x}\right)=\left(\dfrac{x}{1-x}\right)^6$

26. $h\left(t^6\right)=\left(t^6\right)^3-5\left(t^6\right)^2+1=t^{18}-5t^{12}+1$

27. $h\left(\dfrac{x}{1-x}\right)=\left(\dfrac{x}{1-x}\right)^3-5\left(\dfrac{x}{1-x}\right)^2+1$

28. $g\left(x^6\right)=\dfrac{x^6}{1-x^6}$

29. $g(t^3-5t^2+1)=\dfrac{t^3-5t^2+1}{1-(t^3-5t^2+1)}$
$=\dfrac{t^3-5t^2+1}{-t^3+5t^2}$

30. $f(x^3-5x^2+1)=(x^3-5x^2+1)^6$

31. $(x+h)^2-x^2=x^2+2xh+h^2-x^2$
$=2xh+h^2$

32. $\dfrac{1}{x+h}-\dfrac{1}{x}=\dfrac{x-x-h}{x(x+h)}=\dfrac{-h}{x^2+xh}$

33. $\dfrac{\left[4(t+h)-(t+h)^2\right]-\left(4t-t^2\right)}{h}$
$=\dfrac{4t+4h-(t^2+2th+h^2)-4t+t^2}{h}$
$=\dfrac{4h-2th-h^2}{h}=\dfrac{h(4-2t-h)}{h}$
$=4-2t-h$

34. $\dfrac{\left[(t+h)^3+5\right]-\left(t^3+5\right)}{h}$
$=\dfrac{t^3+3t^2h+3th^2+h^3+5-t^3-5}{h}$
$=\dfrac{3t^2h+3th^2+h^3}{h}=\dfrac{h(3t^2+3th+h^2)}{h}$
$=3t^2+3th+h^2$

35. a. $C\left(A(t)\right)=3000+80\left(20t-\dfrac{1}{2}t^2\right)$
$=3000+1600t-40t^2$

b. $C(2)=3000+1600(2)-40(2)^2$
$=3000+3200-160=\$6040$

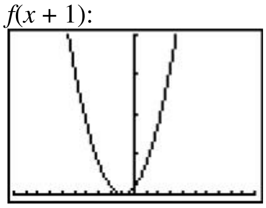
36. a. $C(f(t))$
$=.1(10t-5)^2+25(10t-5)+200$
$=.1(100t^2-100t+25)+250t-125+200$
$=10t^2+240t+77.5$

b. $C(4)=10(4)^2+240(4)+77.5=\$1197.50$

37. $h(x)=f(8x+1)=\left(\dfrac{1}{8}\right)(8x+1)=x+\dfrac{1}{8}$

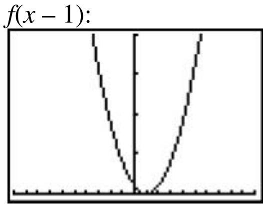
$h(x)$ converts from British to U.S. sizes.

38. $f(x+1)$:



[−10, 10] by [0, 20]

$f(x-1)$:

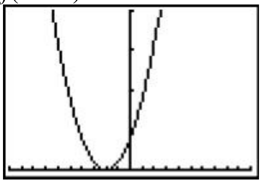


[−10, 10] by [0, 20]

(*continued on next page*)

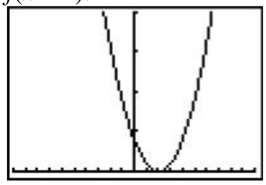
(*continued*)

$f(x+2)$:



[−10, 10] by [0, 20]

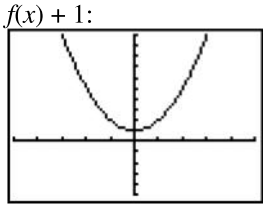
$f(x-2)$:



[−10, 10] by [0, 20]

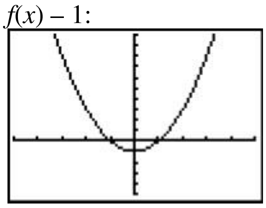
The graph of $f(x+a)$ is the graph of $f(x)$ shifted to the left (if $a > 0$) or to the right (if $a < 0$) by $|a|$ units.

39. $f(x)+1$:



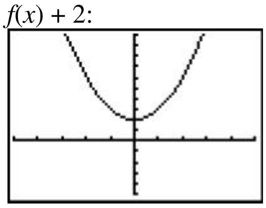
[−5, 5] by [−5, 15]

$f(x)-1$:



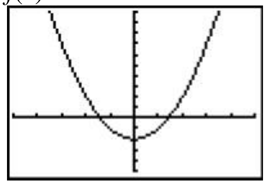
[−5, 5] by [−5, 15]

$f(x)+2$:



[−5, 5] by [−5, 15]

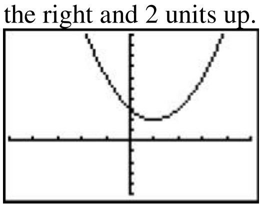
$f(x)-2$:



[−5, 5] by [−5, 15]

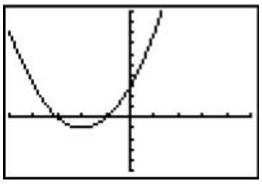
The graph of $f(x)+c$ is the graph of $f(x)$ shifted up (if $c > 0$) or down (if $c < 0$) by $|c|$ units.

40. This is the graph of $f(x)=x^2$ shifted 1 unit to the right and 2 units up.



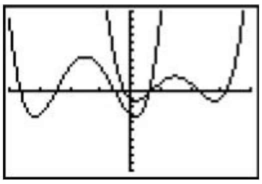
[−5, 5] by [−5, 15]

41. This is the graph of $f(x)=x^2$ shifted 2 units to the left and 1 unit down.



[−5, 5] by [−5, 15]

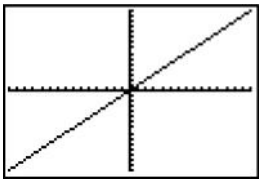
42.



[−4, 4] by [−10, 10]

They are not the same function.

43.



[−15, 15] by [−10, 10]

$$f(f(x)) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$$

$$= \frac{x}{x-(x-1)} = x,\ x \neq 1$$

0.4 Zeros of Functions—The Quadratic Formula and Factoring

1. $f(x) = 2x^2 - 7x + 6$

$2x^2 - 7x + 6 = 0$
$a = 2, b = -7, c = 6$

$\sqrt{b^2 - 4ac} = \sqrt{49 - 4(2)(6)} = \sqrt{1} = 1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{7 \pm 1}{4} = 2, \dfrac{3}{2}$

2. $f(x) = 3x^2 + 2x - 1$

$3x^2 + 2x - 1 = 0$
$a = 3, b = 2, c = -1$

$\sqrt{b^2 - 4ac} = \sqrt{4^2 - 4(3)(-1)} = \sqrt{16} = 4$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2 \pm 4}{6} = \dfrac{1}{3}, -1$

3. $f(t) = 4t^2 - 12t + 9$

$4t^2 - 12t + 9 = 0$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{12 \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$

$= \dfrac{12 \pm \sqrt{0}}{8} = \dfrac{3}{2}$

4. $f(x) = \dfrac{1}{4}x^2 + x + 1$

$\dfrac{1}{4}x^2 + x + 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-1 \pm \sqrt{1^2 - 4\left(\frac{1}{4}\right)(1)}}{2\left(\frac{1}{4}\right)}$

$= \dfrac{-1 \pm \sqrt{0}}{\frac{1}{2}} = -2$

5. $f(x) = -2x^2 + 3x - 4$

$-2x^2 + 3x - 4 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-3 \pm \sqrt{3^2 - 4(-2)(-4)}}{2(-2)}$

$= \dfrac{-3 \pm \sqrt{-23}}{-4}$

$\sqrt{-23}$ is undefined, so $f(x)$ has no real zeros.

6. $f(a) = 11a^2 - 7a + 1$

$11a^2 - 7a + 1 = 0$

$a = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{7 \pm \sqrt{(-7)^2 - 4(11)(1)}}{2(11)}$

$= \dfrac{7 \pm \sqrt{5}}{22} = \dfrac{7 + \sqrt{5}}{22}, \dfrac{7 - \sqrt{5}}{22}$

7. $5x^2 - 4x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{(-4)^2 - 4(5)(-1)}}{2(5)}$

$= \dfrac{4 \pm \sqrt{36}}{10} = \dfrac{4 \pm 6}{10} = 1, -\dfrac{1}{5}$

8. $x^2 - 4x + 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{-4}}{2}$

$\sqrt{-4}$ is undefined, so there is no real solution.

9. $15x^2 - 135x + 300 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{135 \pm \sqrt{(-135)^2 - 4(15)(300)}}{2(15)}$

$= \dfrac{135 \pm \sqrt{225}}{30} = \dfrac{135 \pm 15}{30} = 5, 4$

10. $z^2 - \sqrt{2}z - \dfrac{5}{4} = 0$

$z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{\sqrt{2} \pm \sqrt{\left(-\sqrt{2}\right)^2 - 4(1)\left(-\frac{5}{4}\right)}}{2(1)} = \dfrac{\sqrt{2} \pm \sqrt{7}}{2}$

$= \dfrac{\sqrt{2} + \sqrt{7}}{2}, \dfrac{\sqrt{2} - \sqrt{7}}{2}$

11. $\dfrac{3}{2}x^2 - 6x + 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{6 \pm \sqrt{(-6)^2 - 4\left(\frac{3}{2}\right)(5)}}{2\left(\frac{3}{2}\right)}$

$= \dfrac{6 \pm \sqrt{6}}{3} = 2 + \dfrac{\sqrt{6}}{3}, 2 - \dfrac{\sqrt{6}}{3}$

12. $9x^2 - 12x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12 \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$$
$$= \frac{12 \pm \sqrt{0}}{18} = \frac{2}{3}$$

13. $x^2 + 8x + 15 = (x+5)(x+3)$

14. $x^2 - 10x + 16 = (x-2)(x-8)$

15. $x^2 - 16 = (x-4)(x+4)$

16. $x^2 - 1 = (x+1)(x-1)$

17. $3x^2 + 12x + 12 = 3\left(x^2 + 4x + 4\right)$
$= 3(x+2)(x+2) = 3(x+2)^2$

18. $2x^2 - 12x + 18 = 2\left(x^2 - 6x + 9\right)$
$= 2(x-3)(x-3) = 2(x-3)^2$

19. $30 - 4x - 2x^2 = -2\left(-15 + 2x + x^2\right)$
$= -2(x-3)(x+5)$

20. $15 + 12x - 3x^2 = -3(-5 - 4x + x^2)$
$= -3(x-5)(x+1)$

21. $3x - x^2 = x(3-x)$

22. $4x^2 - 1 = (2x+1)(2x-1)$

23. $6x - 2x^3 = -2x(x^2 - 3)$
$= -2x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$

24. $16x + 6x^2 - x^3 = x\left(16 + 6x - x^2\right)$
$= x(8-x)(x+2)$
$= -x(x-8)(x+2)$

25. $x^3 - 1 = (x-1)\left(x^2 + x + 1\right)$

26. $x^3 + 125 = (x+5)\left(x^2 - 5x + 25\right)$

27. $8x^3 + 27 = (2x+3)\left(4x^2 - 6x + 9\right)$

28. $x^3 - \frac{1}{8} = \left(x - \frac{1}{2}\right)\left(x^2 + \frac{x}{2} + \frac{1}{4}\right)$

29. $x^2 - 14x + 49 = (x-7)^2$

30. $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

31. $2x^2 - 5x - 6 = 3x + 4$
$2x^2 - 8x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(-10)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{144}}{4} = \frac{8 \pm 12}{4} = 5, -1$$

$y = 3x + 4 = 15 + 4 = 19$
$y = -3 + 4 = 1$
Points of intersection: (5, 19), (–1, 1)

32. $x^2 - 10x + 9 = x - 9$
$x^2 - 11x + 18 = 0$
$(x-9)(x-2) = 0$
$x = 9, 2$
$y = x - 9 = 9 - 9 = 0$
$y = 2 - 9 = -7$
Points of intersection: (9, 0), (2, –7)

33. $y = x^2 - 4x + 4$
$y = 12 + 2x - x^2$
$x^2 - 4x + 4 = 12 + 2x - x^2$
$2x^2 - 6x - 8 = 0$
$2(x^2 - 3x - 4) = 0$
$2(x-4)(x+1) = 0$
$x = 4, -1$
$y = x^2 - 4x + 4 = 4^2 - 4(4) + 4 = 4$
$y = (-1)^2 - 4(-1) + 4 = 9$
Points of intersection: (4, 4), (–1, 9)

34. $y = 3x^2 + 9$
$y = 2x^2 - 5x + 3$
$3x^2 + 9 = 2x^2 - 5x + 3$
$x^2 + 5x + 6 = 0$
$(x+3)(x+2) = 0$
$x = -3, -2$
$y = 3x^2 + 9 = 3(-3)^2 + 9 = 36$
$y = 3(-2)^2 + 9 = 21$
Points of intersection: (–3, 36), (–2, 21)